CH6605 Process Instrumentation, Dynamics and Control

Bode Plots

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Introduction

- The Bode plots (in honor of H. W. Bode) constitute a convenient way to represent the frequency response characteristics of a system.
- The amplitude ratio (AR) and the phase shift (ϕ) of the ultimate response of a system are functions of the frequency ω.

Introduction

Bode plots consist of a pair of the plots showing:

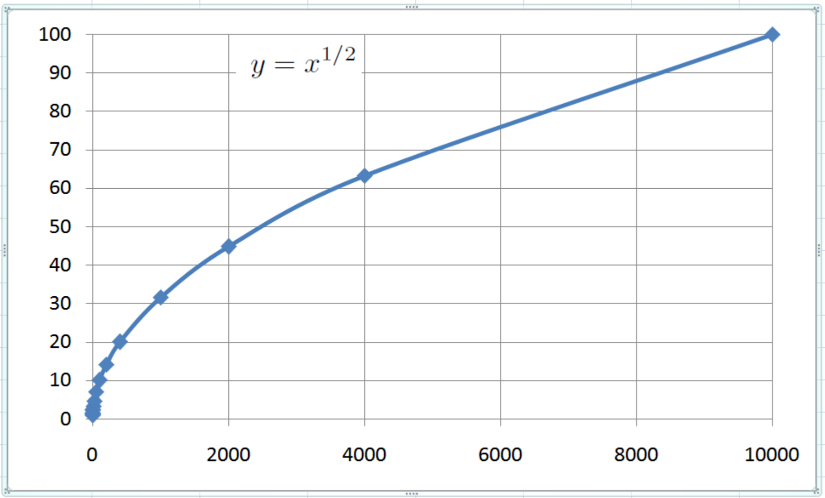
- How the logarithm of the amplitude ratio varies with frequency.
- How the phase shift varies with frequency.

In order to cover a large range of frequencies, we use logarithmic scale for frequencies.

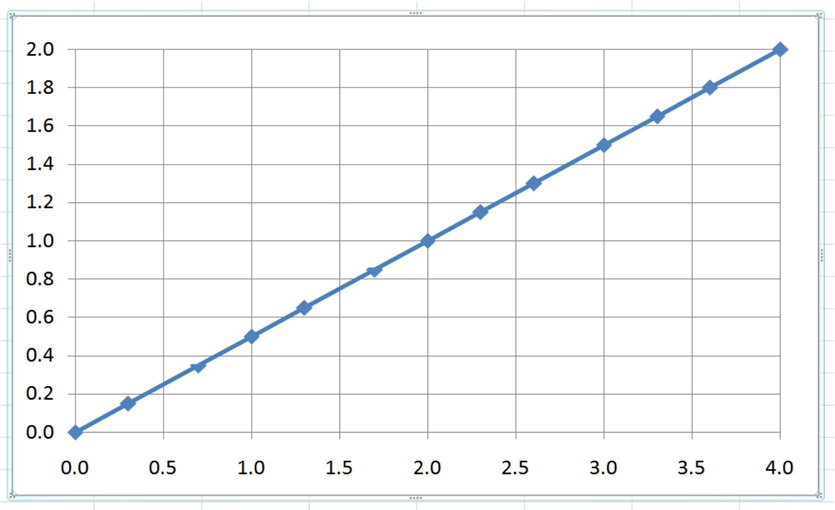
Plot of $\sqrt{x}$ vs. x

	y=sqrt(x)		
x	y	log x	log y
1	1.0	0.0	0.00
2	1.4	0.3	0.15
5	2.2	0.7	0.35
10	3.2	1.0	0.50
20	4.5	1.3	0.65
50	7.1	1.7	0.85
100	10.0	2.0	1.00
200	14.1	2.3	1.15
400	20.0	2.6	1.30
1000	31.6	3.0	1.50
2000	44.7	3.3	1.65
4000	63.2	3.6	1.80
10000	100.0	4.0	2.00

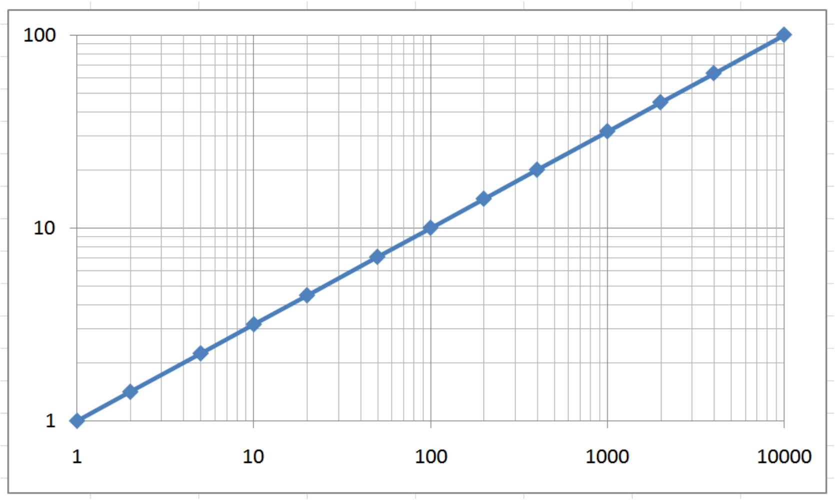
Plot of $\sqrt{x}$ vs. x — Linear Scales



Plot of $\log \sqrt{x}$ vs. $\log x$ — Linear Scales



Plot of $\sqrt{x}$ vs. x — Log-Log Scales



Bode Plots of First Order System

For first order system,

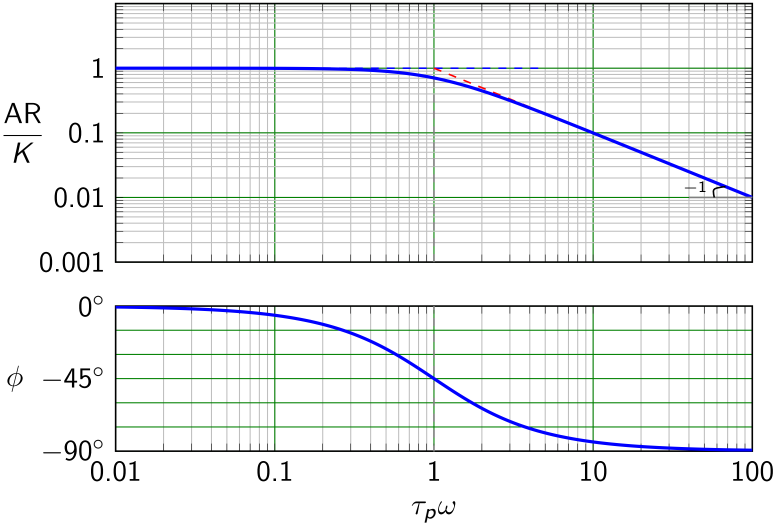
$$\text{AR} = \frac{K_p}{\sqrt{1 + \tau_p^2\omega^2}}$$

$$\phi = \tan^{-1}(-\tau_p\omega)$$

$$\log\left(\frac{\text{AR}}{K_p}\right) = -\frac{1}{2}\log(1 + \tau_p^2\omega^2)$$

The plot of $\log(\text{AR}/K_p)$ versus $\log(\tau_p\omega)$ is shown in figure (given in the following page) as a solid (blue) line.
Instead of the very elaborate numerical work needed to plot this graph, we can give an approximate sketch by considering its asymptotic behavior as $\omega \to 0$ and as $\omega \to \infty$.

Bode Plots of First Order System



Asymptotic Plots for First Order System

$$\log\left(\frac{\text{AR}}{K_p}\right) = -\frac{1}{2}\log(1+\tau_p^2\omega^2) \tag{1}$$

1. As $\omega \to 0$, then $\tau_p\omega \to 0$, and from Eqn.(1), $\log\left(\frac{\text{AR}}{K_p}\right) \to 0$ or $\left(\frac{\text{AR}}{K_p}\right) \to 1$. This is the low-frequency asymptote shown by a blue dashed line.
2. As $\omega \to \infty$, then $\tau_p\omega \to \infty$, and from Eqn.(1), $\log\left(\frac{\text{AR}}{K_p}\right) \approx -\log(\tau_p\omega)$. This is the high-frequency asymptote shown by a red dashed line. It is a line with a slope of -1 passing through the point $\frac{\text{AR}}{K_p} = 1$ for $\tau_p\omega = 1$.

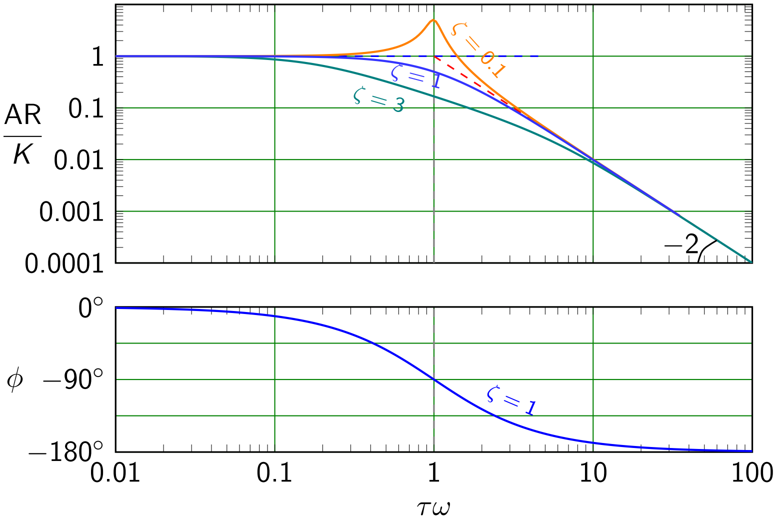
Asymptotic Plots for First Order System (contd..)

The frequency $\omega = 1/\tau_p$ is known as the corner frequency. At the corner frequency, the deviation of the true value of AR/K_p from the asymptotes is maximum.

The plot of ϕ vs. $\tau_p\omega$ is constructed from the following characteristics:

- As $\omega \to 0$, then $\phi \to 0$.
- As $\omega \to \infty$, then $\tan^{-1}(-\infty) = -90°$.
- At $\omega = 1/\tau_p$ (corner frequency), $\phi = \tan^{-1}(-1) = -45°$.

Bode Plots of Second Order System



Second Order System - Asymptotic Plots

$$\text{AR} = \frac{K_p}{\sqrt{(1-\tau^2\omega^2)^2 + (2\zeta\tau\omega)^2}}$$

$$\phi = \tan^{-1}\left(-\frac{2\zeta\tau\omega}{1-\tau^2\omega^2}\right)$$

- As $\omega \to 0$, then $\log\left(\frac{\text{AR}}{K_p}\right) \to 0$ or $\left(\frac{\text{AR}}{K_p}\right) \to 1$.
- As $\omega \to \infty$, then $\log\left(\frac{\text{AR}}{K_p}\right) \to -2\log(\tau\omega)$. It is a straight line with a slope of -2 passing through the point $\log\left(\frac{\text{AR}}{K_p}\right) = 1$, when $\tau\omega = 1$

Bode Plot of Pure Capacitive Process

Pure capacitive process (also known as integrating system) has the following transfer function:

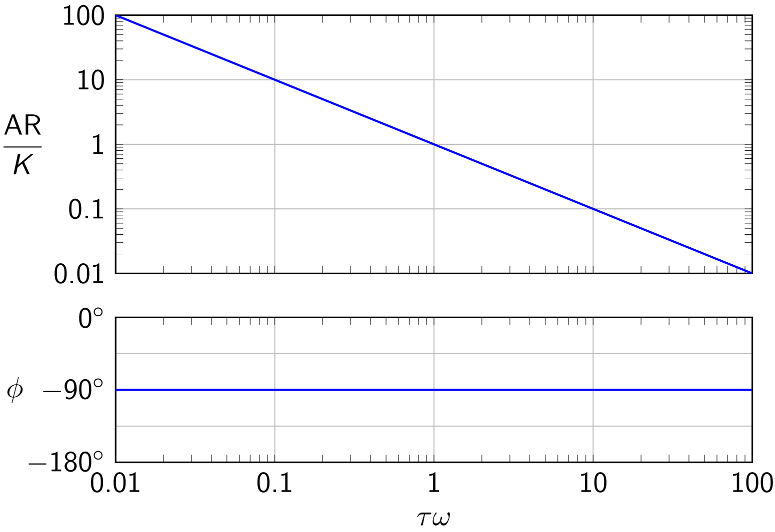
$$G(s) = \frac{K_p}{s}$$

By substituting $s = j\omega$, we get

$$G(j\omega) = \frac{K_p}{j\omega} = \frac{K_p}{j\omega}\frac{j\omega}{j\omega} = -j\frac{K_p}{\omega}$$

$$\text{AR} = \sqrt{0^2 + (-Kp/\omega)^2} = \frac{K_p}{\omega} \qquad \text{and} \qquad \phi = \tan^{-1} -\infty = -90^\circ$$

Bode Plot of Pure Capacitive Process (contd..)



Bode Plot of Dead-time System

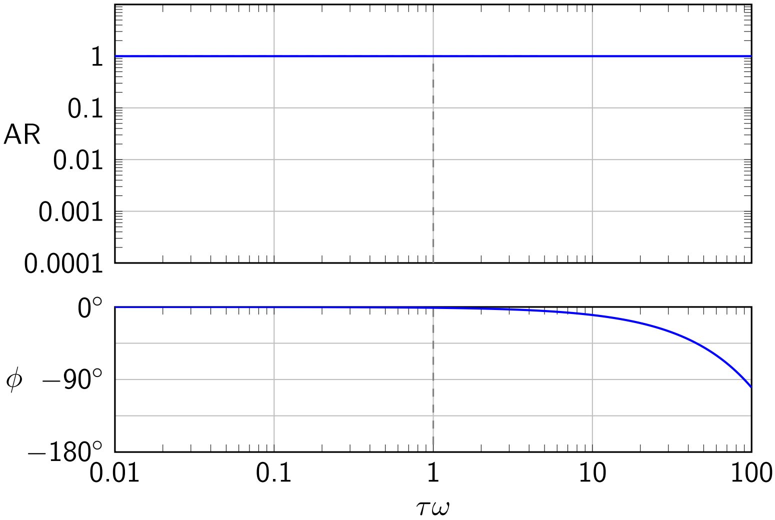
For the dead-time system,

$$Y(t) = X(t - \tau_d) \qquad G(s) = \frac{Y(s)}{X(s)} = e^{-\tau_d s}$$

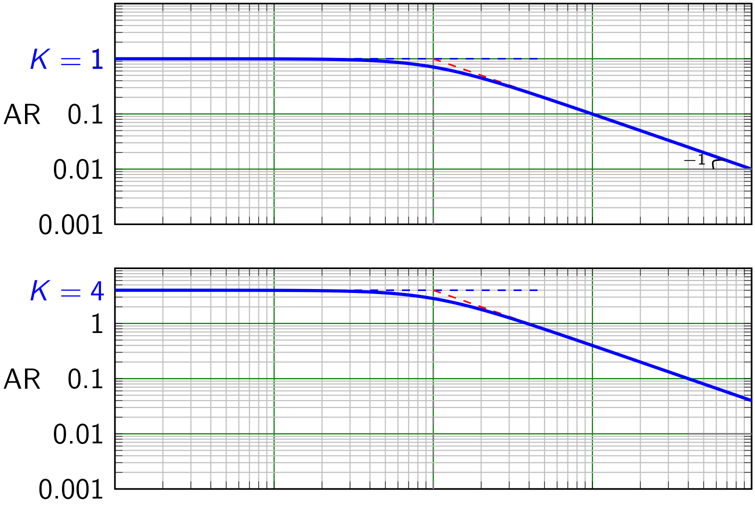
Amplitude ratio (AR) and phase shift (ϕ) of dead-time system is given by

$$\text{AR} = 1 \qquad \phi = -\tau_d \omega$$

Bode Plot of Dead-time System (contd..)



Effect of K in Bode Plot for $G(s) = \dfrac{K}{\tau s + 1}$



Effect of K in Bode Plot

The presence of a constant in the overall transfer function will move the entire AR curve vertically by a constant amount. It has no effect on the phase shift.

Systems in Series

The overall transfer function is:

$$G(s) = G_1(s)G_2(s)\cdots G_N(s)$$

Overall phase shift ϕ:

$$\phi = \phi_1 + \phi_2 + \cdots + \phi_N$$

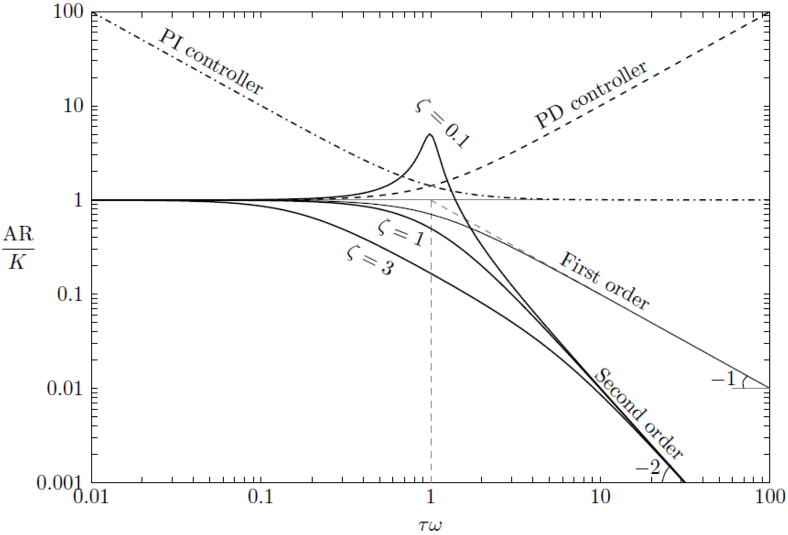
Overall amplitude ratio:

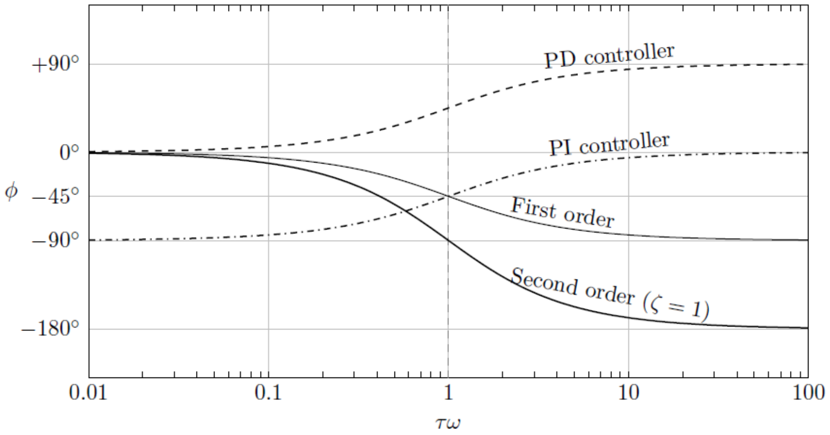
$$\begin{aligned} \mathrm{AR} &= (\mathrm{AR})_1(\mathrm{AR})_2\cdots(\mathrm{AR})_N \\ \log(\mathrm{AR}) &= \log(\mathrm{AR})_1 + \log(\mathrm{AR})_2 + \cdots + \log(\mathrm{AR})_N \end{aligned}$$

Systems in Series (contd..)

If the transfer function of a system can be factored into the product of N transfer functions of simpler systems, the following rules can be used for the construction of Bode plots:

- The logarithm of the overall amplitude ratio is equal to the sum of the logarithms of the amplitude rations of the individual systems.
- The overall phase shift is equal to the sum of the phase shifts of the individual systems.
- The presence of a constant in the overall transfer function will move the entire AR curve vertically by a constant amount. It has no effect on the phase shift.


PI controller
PD controller
$\zeta = 0.1$
$\zeta = 1$
$\zeta = 3$
First order
Second order
-1
-2
100
10
1
0.1
0.01
0.001
$\frac{\mathrm{AR}}{K}$
0.01
0.1
1
10
100
$\tau\omega$



PD controller
PI controller
First order
Second order ($\zeta = 1$)
$+90°$
$0°$
$-45°$
$-90°$
$-180°$
ϕ
0.01
0.1
1
10
100
$\tau\omega$

$$\text{Bode Plot of: } G = \frac{3(s+1)}{(0.05s+1)(10s+1)}$$

Constructing the Bode Plot: Transfer function of the given system is factored into a product of four transfer functions as below:

$$3, \qquad \frac{1}{10s+1}, \quad (s+1), \quad \frac{1}{0.05s+1}$$

with the following corner frequencies (in the same order):

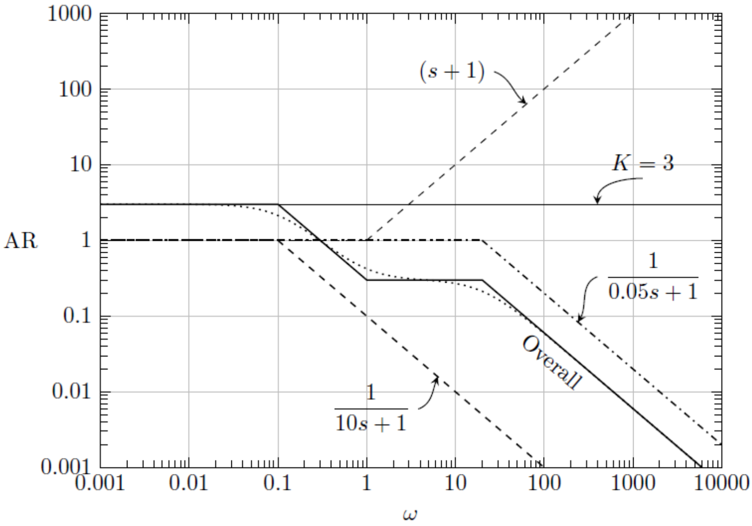
$$\omega_1 = \text{none}, \qquad \omega_2 = 1/10 = 0.1, \qquad \omega_3 = 1/1 = 1, \qquad \omega_4 = 1/0.05 = 20$$

The following four regions are identified on the frequency (in ω) scale:

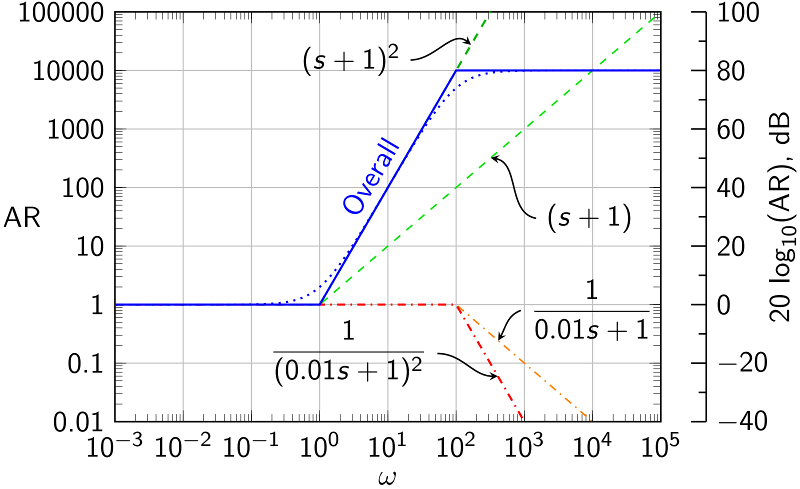
$$0 \le \omega \le \omega_2, \qquad \omega_2 \le \omega \le \omega_3, \qquad \omega_3 \le \omega \le \omega_4, \qquad \omega_4 \le \omega < \infty$$

$$0 \le \omega \le 0.1, \qquad 0.1 \le \omega \le 1, \qquad 1 \le \omega \le 20, \qquad 20 \le \omega < \infty$$

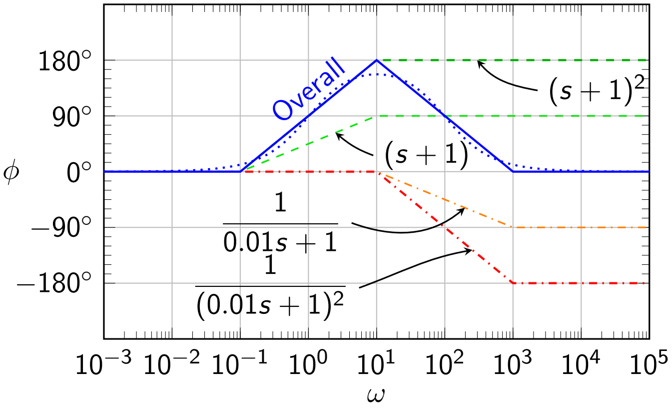
Frequency region	Slopes of the asymptotes of the transfer functions				
	$K = 3$	$\frac{1}{10s+1}$	$(s+1)$	$\frac{1}{0.05s+1}$	Overall
$0 \le \omega \le 0.1$	0	0	0	0	0
$0.1 \le \omega \le 1$	0	-1	0	0	-1
$1 \le \omega \le 20$	0	-1	1	0	0
$20 \le \omega < \infty$	0	-1	1	-1	-1


1000
100
10
1
0.1
0.01
0.001
AR
0.001
0.01
0.1
1
10
100
1000
10000
ω
$(s+1)$
$K=3$
$\frac{1}{0.05s+1}$
$\frac{1}{10s+1}$
Overall

Bode Plot of $G = 10^4 \dfrac{(s+1)^2}{(s+100)^2}$



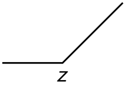
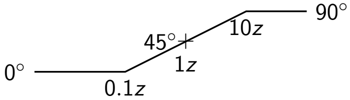
Bode Plot of $G = 10^4 \dfrac{(s+1)^2}{(s+100)^2}$ (contd..)



The transfer function is rewritten as:

$$G = 10^4 \cdot (s+1) \cdot (s+1) \cdot \frac{1}{100\left(\frac{s}{100}+1\right)} \cdot \frac{1}{100\left(\frac{s}{100}+1\right)}$$

$$= (s+1) \cdot (s+1) \cdot \frac{1}{(0.01s+1)} \cdot \frac{1}{(0.01s+1)}$$

Transfer function	Gain Plot (slope of ± 1)	Phase Plot
Zero, of the form: $\left(\frac{s}{z}+1\right)$		
Pole, of the form: $\frac{1}{\left(\frac{s}{p}+1\right)}$