Heat Transfer

Conduction - Multiple Resistances

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Objectives

- To understand and solve problems of steady state heat conduction with multiple resistances.

Outcome

- To solve the problems of steady state heat conduction with multiple resistances.

Heat Conduction with Multiple Resistances

- Composites are formed by bringing together into close contact distinct materials. The material properties of the assembly change discontinuously at the contact surface. Also, the mechanical conditions of the contact at the interface affect the rate at which energy flows across it.
- The idea of thermal resistance is a useful tool for analyzing conduction through composite members.

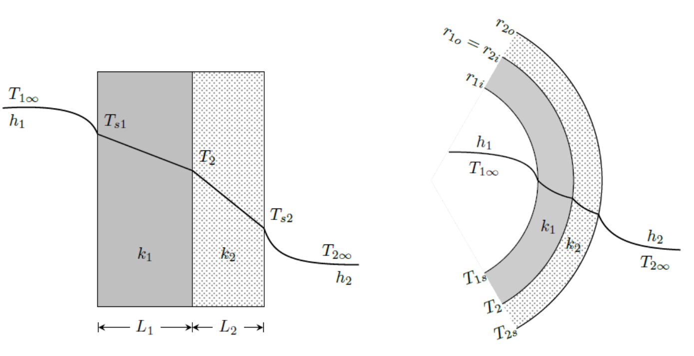
The idea of resistance was already introduced and discussed for conduction.
The same approach can be used for convection as well, as below:

$$Q = hA(T_s - T_\infty) = \frac{(T_s - T_\infty)}{R}$$

where

$$R = \frac{1}{hA}$$

Composite Wall
Flat Wall



(a) Flat Surface (b) Curved Surface

$$Q = \frac{T_{\infty,1} - T_{s,1}}{R_1} = \frac{T_{s,1} - T_2}{R_2} = \frac{T_2 - T_{s,2}}{R_3} = \frac{T_{s,2} - T_{\infty,2}}{R_4}$$

where

$$R_1 = \frac{1}{h_1 A} \qquad R_2 = \frac{L_1}{k_1 A} \qquad R_3 = \frac{L_2}{k_2 A} \qquad R_4 = \frac{1}{h_2 A}$$

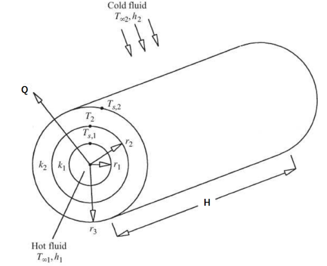
Adding the numerators and denominators separately, we get

$$Q = \frac{T_{\infty_1} - T_{\infty,2}}{R_1 + R_2 + R_3 + R_4}$$

From the rule of proportions,

$$\frac{a}{b} = \frac{c}{d} \qquad \Longrightarrow \qquad \frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$$

Composite Hollow Cylinder

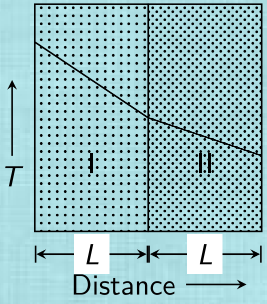


$$Q = \frac{T_{\infty_1} - T_{\infty,2}}{R_1 + R_2 + R_3 + R_4}$$

where

$$R_1 = \frac{1}{h_1 2\pi r_1 H} \quad R_2 = \frac{\ln(r_2/r_1)}{2\pi k_1 H} \quad R_3 = \frac{\ln(r_3/r_2)}{2\pi k_2 H} \quad R_4 = \frac{1}{h_2 2\pi r_2 H}$$

At steady state, the temperature variation in a plane wall, made of two different solids I and II is shown below:



Then, the thermal conductivity of material I (G-1997-2.09)

(a) is smaller than that of II
(b) is greater than that of II
(c) is equal to that of II
(d) can be greater than or smaller than that of II

(a) ✓ *Explanation:* $q = k\dfrac{\Delta T}{\Delta x} = k \times \text{slope} = \text{const.}$ Here, $\text{slope}_{\text{I}} > \text{slope}_{\text{II}}$. Therefore $k_{\text{I}} < k_{\text{II}}$. □

Example 1: Conduction followed by Convection

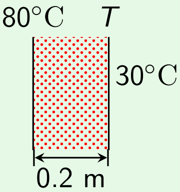
The left face of a one dimensional slab of thickness 0.2 m is maintained at $80^\circ\mathrm{C}$ and the right face is exposed to air at $30^\circ\mathrm{C}$. The thermal conductivity of the slab is 1.2 W/(m.K) and the heat transfer coefficient from the right face is 10 W/(m^2.K). At steady state. the temperature of the right face in $^\circ\mathrm{C}$ is (G-2004-58)

(a) 77.2 (b) 71.2 (c) 63.8 (d) 48.7

Solution:

(d) ✓ *Explanation:*

At steady
state, rate of heat transfer by conduction through the slap is equal to the rate of heat transfer by convection in the air. i.e.,



$$k\,\frac{\Delta T_1}{\Delta x} = h\,\Delta T_2$$

Substituting the known data, we get

$$\frac{1.2 \times (80 - T)}{0.2} = 10 \times (T - 30)$$

Solving, we get $T = 48.7^{\circ}\mathrm{C}$. □

Example 2: Composite Wall

The inner wall of a furnace is at a temperature of 700°C. The composite wall is made of two substances, 10 and 20 cm thick with thermal conductivities of 0.05 and 0.1 W/m.$^\circ$C respectively. The ambient air is at 30°C and the heat transfer coefficient between the outer surface of wall and air is 20 W/m^2.$^\circ$C. The rate of heat loss from the outer surface in W/m^2 is (G-2003-57)

(a) 165.4 (b) 167.5 (c) 172.8 (d) 175

Solved Problems (contd..)

Solution:

(a) ✓ *Explanation:* $q = \dfrac{\Delta T}{R}$, where

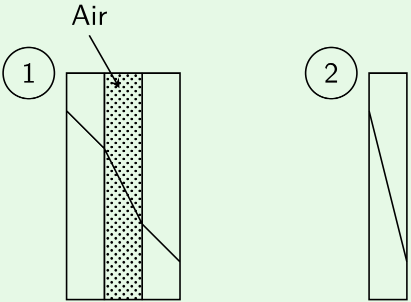
$$R = \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{1}{h} = \frac{0.1}{0.05} + \frac{0.2}{0.1} + \frac{1}{20} = 4.05^\circ\mathrm{C.m^2/W}$$

Therefore, $q = \dfrac{\Delta T}{R} = \dfrac{(700 - 30)}{4.05} = 165.4\ \mathrm{W/m^2}$ □

Example 3: Heat Loss Reduction by Thermopane

A thermopane window consists of two sheets of glass each 6 mm thick, separated by a layer of stagnant air also 6 mm thick. Find the percentage reduction in heat loss from this pane as compared to that of a single sheet of glass 6 mm thickness. The temperature drop between inside and outside remains same at $15°\mathrm{C}$. Thermal conductivity of glass is 30 times that of air. (G-1988-14.iii)

Solution:



$$q = \frac{\Delta T}{L/k}$$

Let k be the thermal conductivity of glass. Then, thermal conductivity of air $= k/30$.

$$\begin{aligned} q_1 &= \frac{15}{\frac{6}{k} + \frac{6 \times 30}{k} + \frac{6}{k}} = \frac{15}{192}k \\ q_2 &= \frac{15}{6/k} = \frac{15}{6}k \end{aligned}$$

$$\begin{aligned} \text{Reduction in heat loss} &= \frac{q_2 - q_1}{q_2} \times 100 \\ &= \frac{(15/6) - (15/192)}{(15/6)} \times 100 = 96.9\% \end{aligned}$$

Example 4: Heat Transfer Through Cylindrical Surface

Heat is generated at a steady rate of 100 W due to resistance heating in a long wire (length $=$ 5 m, diameter $=$ 2 mm). This wire is wrapped with an insulation of thickness 1 mm that has a thermal conductivity of 0.1 W/m.K. The insulated wire is exposed to air at $30^{\circ}\mathrm{C}$. The convective heat transfer between the wire and surrounding air is characterized by a heat transfer coefficient of 10 W/m^2.K. The temperature (in $^{\circ}\mathrm{C}$) at the interface between the wire and the insulation is (G-2012-34)

(a) 211.2 (b) 242.1 (c) 311.2 (d) 484.2

Solution:

Rate of heat transfer from a cylindrical surface is given by

$$Q = \frac{\Delta T}{R} = \frac{T_1 - T_2}{\left(\frac{t}{kA_m}\right)_{\text{ins}} + \left(\frac{1}{h_2 A_2}\right)}$$

where

$$\begin{aligned} t &= (r_2 - r_1), \text{ thickness of insulation, and} \\ A_m &= A_{lm} = \frac{A_2 - A_1}{\ln(A_2/A_1)} = \frac{2\pi(r_2 - r_1)L}{\ln(r_2/r_1)} \end{aligned}$$

Therefore,

$$A_m = \frac{2\pi(r_2 - r_1)L}{\ln(r_2/r_1)} = \frac{2 \times \pi \times (0.002 - 0.001) \times 5}{\ln(0.002/0.001)} = 0.0453 \text{ m}^2$$

Using this and subsituting the known values in the equation for Q, we get

$$100 = \frac{T_1 - 30}{\left(\frac{0.001}{0.1 \times 0.0453}\right) + \left(\frac{1}{10 \times (2\pi \times 0.002 \times 5)}\right)}$$

Solving the above, we get $T_1 = 211.2°\mathrm{C}$. **(a)** ✓

Questions for Practice

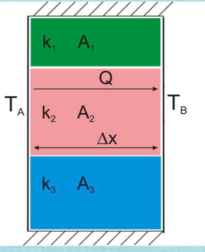
1. The wall of a building is a multi-layered composite consisting of brick (100 mm layer), a 100 mm layer of glass fiber (28 kg/m^3), a 10 mm layer of gypsum plaster, and a 6 mm layer of pine panel. If h_{inside} is 10 W/(m^2.K) and h_{outside} is 70 W/(m^2.K), calculate the total thermal resistance for heat transfer.
   Thermal conductivity of materials: Brick, $k_b = 1.3$ W/(m.K); Glass fiber (28kg/m3), $k_{gl} = 0.038$ W/(m.K); Gypsum, $k_{gy} = 0.17$ W/(m.K): Pine panel, $k_p = 0.12$ W/(m.K).
   (Ans: 2.93 m^2.K/W)

2. A 1m long metal plate with thermal conductivity $k = 50$ W/(m.K) is insulated on its sides. The top surface is maintained at $100^{\circ}\mathrm{C}$ while the bottom surface is convectively cooled by a fluid at $20^{\circ}\mathrm{C}$. Under steady state conditions and with no volumetric heat generation, the temperature at the midpoint of the plate is measured to be $85^{\circ}\mathrm{C}$. Calculate the value of the convection heat transfer coefficient at the bottom surface. (Ans: 30 W/(m^2.K))
3. A steam pipe of 0.12 m outside diameter is insulated with a 20 mm thick layer of calcium silicate, with $k = 0.089$ W/(m.K). If the inner and outer surfaces of the insulation are at temperatures of $T_1 = 800$ K and $T_2 = 490$ K, respectively, what is the heat loss per unit length of the pipe? (Ans: 603 W/m)

4. A cylindrical nuclear fuel rod of 0.2 m dia has a thermal conductivity $k = 0.5$ W/(m.K) and generates uniformly 24,000 W/m^3. This rod is encapsulated within another cylinder having an outer radius of 0.2 m and a thermal conductivity of 4 W/(m.K). The outer surface is cooled by a coolant fluid at $100^\circ\mathrm{C}$, and the convection coefficient between the outer surface and the coolant is 20 W/(m^2.K). Find the temperatures at the interface between the two cylinders and at the outer surface. (Ans: $T_i = 151.5^\circ\mathrm{C}$; $T_s = 130.4^\circ\mathrm{C}$)

5. Steam at $T_{\infty,1} = 320°\mathrm{C}$ flows in a cast iron pipe [$k = 80$ W/(m.$°\mathrm{C}$)] whose inner and outer diameter are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with a 3 cm thick glass wool insulation [$k = 0.05$ W/(m.$°\mathrm{C}$]. Heat is lost to the surroundings at $T_{\infty,2} = 5°\mathrm{C}$ by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/(m^2.$°\mathrm{C}$. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/(m^2.K), determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drop across the pipe shell and the insulation. (Ans: $Q = 120.7$ W/m; $\Delta T_{\text{pipe}} = 0.02°\mathrm{C}$; $\Delta T_{\text{insulation}} = 284°\mathrm{C}$)

Resistances in Parallel



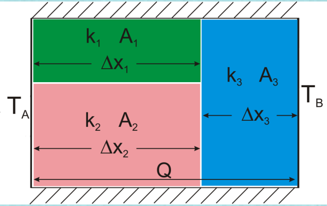
$$Q = Q_1 + Q_2 + Q_3 = \frac{T_A - T_B}{R_1} + \frac{T_A - T_B}{R_2} + \frac{T_A - T_B}{R_3}$$

$$= (T_A - T_B)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$= \frac{T_A - T_B}{R}$$

where

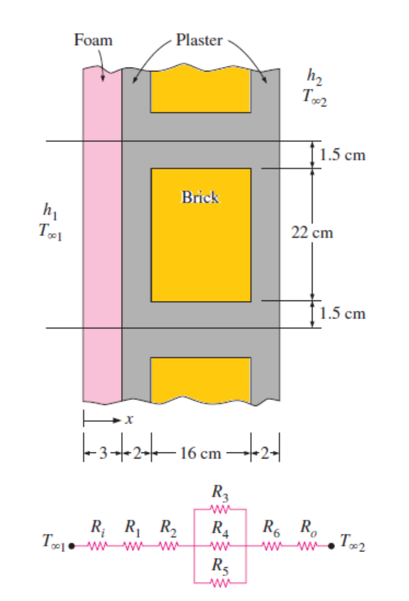
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

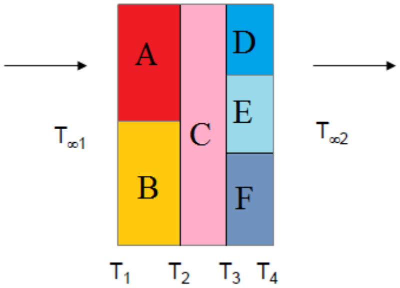


$$Q = \frac{T_A - T_B}{R_{\text{total}}}$$

$$\begin{aligned} R_{\text{total}} &= R_{(1 \text{ and } 2)} + R_3 \\ &= \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} + R_3 \\ &= \frac{R_1 R_2}{R_1 + R_2} + R_3 \end{aligned}$$

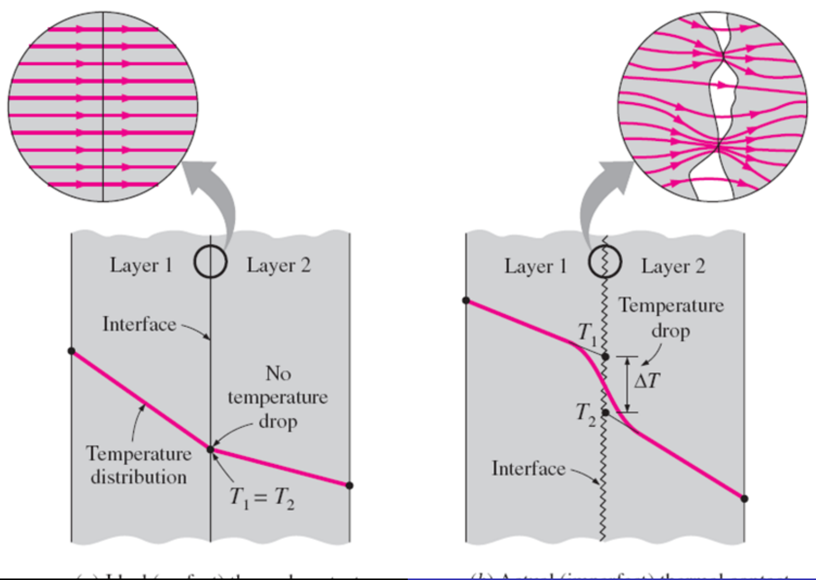
Resistances in Series and Parallel





$$R = R_{\infty 1} + \frac{1}{\left(\frac{1}{R_A} + \frac{1}{R_B}\right)} + R_C + \frac{1}{\left(\frac{1}{R_D} + \frac{1}{R_E} + \frac{1}{R_F}\right)} + R_{\infty 2}$$

Contact Resistance



- When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air. These numerous air gaps of varying sizes act as insulation because of the low thermal conductivity of air. Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the thermal contact resistance, R_C.
- The value of thermal contact resistance depends on: surface roughness, material properties, temperature and pressure at the interface type of fluid trapped at the interface.
- Thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be disregarded for poor heat conductors such as insulations.

The inner wall of a furnace is at a temperature of 700°C. The composite wall is made of two substances, 10 and 20 cm thick with thermal conductivities of 0.05 and 0.1 W/m.°C respectively. The ambient air is at 30°C and the heat transfer coefficient between the outer surface of wall and air is 20 $W/m^2.°C$. The rate of heat loss from the outer surface in W/m^2 is (G-2003-57)

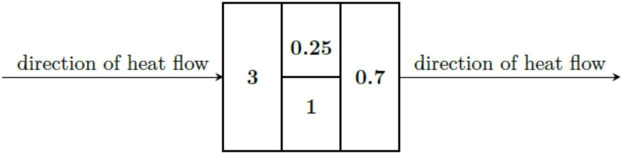
(a) 165.4 (b) 167.5 (c) 172.8 (d) 175

(a) ✓ *Explanation:* $q = \dfrac{\Delta T}{R}$, where

$$R = \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{1}{h} = \frac{0.1}{0.05} + \frac{0.2}{0.1} + \frac{1}{20} = 4.05^\circ\mathrm{C.m^2/W}$$

Therefore, $q = \dfrac{\Delta T}{R} = \dfrac{(700-30)}{4.05} = 165.4\ \mathrm{W/m^2}$

A composite wall is made of four different materials of construction in the fashion shown below. The resistance (in K/W) of each of the sections of the wall is indicated in the diagram.



The overall resistance (in K/W, rounded off to the first decimal place) of the composite wall, in the direction of heat flow, is ________ (G-2016-9)

(3.9) ✓ *Explanation:* Total resistance of the composite wall is given by

$$R_{\text{overall}} = R_1 + R_{23} + R_4$$

Here, materials 2 and 3 are in parallel. Combined resistance of them is given by

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 + R_3}{R_2 R_3} \qquad \Longrightarrow \qquad R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$

Hence,

$$R_{\text{overall}} = R_1 + R_{23} + R_4 = R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4 = 3 + \frac{0.25 \times 1}{0.25 + 1} + 0.7 = 3.9 \text{ K/W} \qquad \square$$

The bottom face of a horizontal slab of thickness 6 mm is maintained at 300°C. The top face is exposed to a flowing gas at 30°C. The thermal conductivity of the slab is 1.5 W/(m.K) and the convective heat transfer coefficient is 30 W/(m^2.K). At steady state, the temperature (in °C) of the top face is ________ (G-2014-47)

(271) ✓ *Explanation:* At steady state, for a flat surface, heat flux remains constant. i.e.,

$$q = \frac{\Delta T}{R} = \frac{\Delta T_1}{R_1}$$

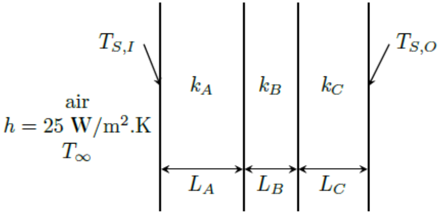
Therefore,

$$\frac{300 - 30}{\frac{x}{k} + \frac{1}{h}} = \frac{300 - T_1}{\frac{x}{k}} \implies \frac{270}{\frac{0.006}{1.5} + \frac{1}{30}} = \frac{300 - T_1}{\frac{0.006}{1.5}} \implies T_1 = 271°\text{C} \quad \square$$

The composite wall of an oven consists of three materials A, B and C. Under steady state operating conditions, the outer surface temperature $T_{S,O}$ is 20°C, the inner surface temperature $T_{S,I}$ is 600°C and the oven air temperature is $T_\infty = 800$°C. For the following data

thermal conductivities $k_A = 20$ W/(m.K) and $k_C = 50$ W/(m.K),
thickness $L_A = 0.3$ m, $L_B = 0.15$ m and $L_C = 0.15$ m
inner-wall heat transfer coefficient $h = 25$ W/(m^2.K),

the thermal conductivity k_B in W/(m.K) of the material B, is calculated as (G-2007-42)



(a) 35 (b) 1.53 (c) 0.66 (d) 0.03

(b) ✓ *Explanation:*

$$q = h(T_\infty - T_{S,I}) = \frac{T_{S,I} - T_{S,O}}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}}$$

Substituting the known quantities, we get

$$25 \times (800 - 600) = \frac{600 - 20}{\dfrac{0.3}{20} + \dfrac{0.15}{k_B} + \dfrac{0.15}{50}}$$

Solving, we get $k_B = 1.53$ W/m.K