Heat Transfer

Conduction - Critical Radius of Insulation

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September 25, 2019

Objectives

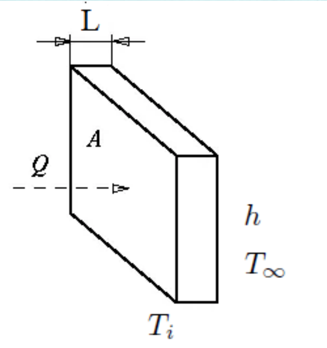
- To understand the issues associate with insulation addition for a curved surface.
- To obtain the equation for critical radius of insulation.
- To give an idea about minimizing the annual cost of insulation.

Outcome

- To understand the effect of curvature in increasing / decreasing the rate of heat transfer.

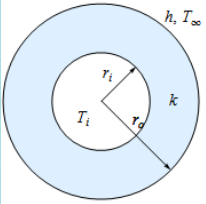
Addition of Insulation to Flat Wall

We know that by adding more insulation to a flat wall always decreases heat transfer, because of the added resistance. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the total resistance for heat transfer.



Insulating Curved Surfaces

Adding insulation to a cylindrical piece or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.



The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

$$Q = \frac{T_i - T_\infty}{R} = \frac{T_i - T_\infty}{R_{\text{ins}} + R_o}$$

where

$$R_{\text{ins}} = \frac{\ln(r_o/r_i)}{2\pi kH} \qquad \text{and} \qquad R_o = \frac{1}{2\pi r_o Hh}$$

Q may have a maximum for a certain value of $r_o = r_{oc}$. This critical value is obtained by differentiating Q with respect to r_o, and setting the resulting expression to zero.

$$Q = \frac{(T_i - T_\infty)2\pi kH}{\ln\left(\frac{r_o}{r_i}\right) + \frac{k}{hr_o}}$$

$$\frac{dQ}{dr_o} = 0 \qquad \Longrightarrow \qquad \frac{d}{dr_o}\left[\ln\left(\frac{r_o}{r_i}\right) + \frac{k}{hr_o}\right] = 0$$

$$\frac{d}{dr_o}\left[\ln\left(\frac{r_o}{r_i}\right)+\frac{k}{hr_o}\right]=\frac{d}{dr_o}\left[\ln r_o-\ln r_i+\frac{k}{hr_o}\right]=\frac{1}{r_o}-\frac{k}{hr_o^2}$$

Equating the above to zero (at $r_o = r_{oc}$), gives

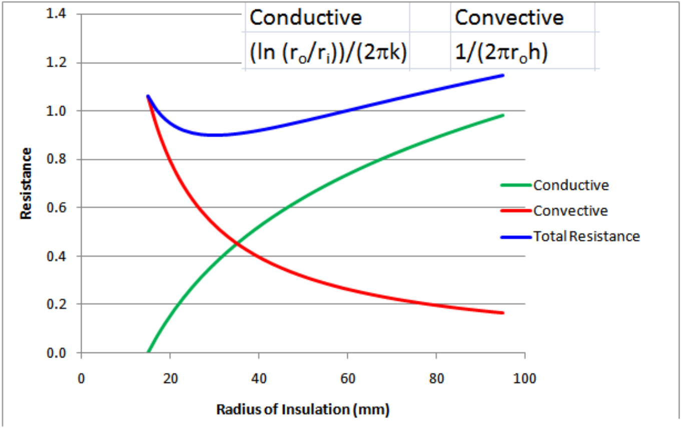
$$\frac{1}{r_{oc}}-\frac{k}{hr_{oc}^2}=0 \qquad \Longrightarrow \qquad \boxed{r_{oc}=\frac{k}{h}}$$

Critical Radius of Insulation (contd..)

Cylindrical surface

k	**0.300**	W/(m.K)	r_c	30.0	mm
h	**10**	$W/(m^2.K)$			
r_i	**15**	mm			

Conductive	Convective
$(\ln(r_o/r_i))/(2\pi k)$	$1/(2\pi r_o h)$



$$Q = \frac{T_i - T_\infty}{R} = \frac{T_i - T_\infty}{R_{\text{ins}} + R_o}$$

where

$$R_{\text{ins}} = \frac{r_o - r_i}{4\pi k r_o r_i} \qquad \text{and} \qquad R_o = \frac{1}{h 4\pi r_o^2}$$

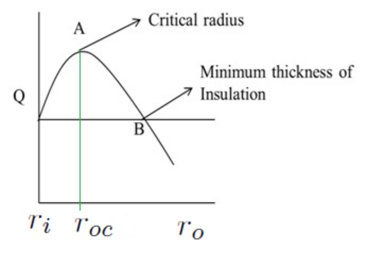
$$Q = \frac{(T_i - T_\infty) 4\pi k}{\left(\frac{r_o - r_i}{r_o r_i}\right) + \frac{k}{h r_o^2}}$$

$$\frac{dQ}{dr_o} = 0 \qquad \Longrightarrow \qquad \frac{d}{dr_o}\left[\left(\frac{r_o - r_i}{r_o r_i}\right) + \frac{k}{h r_o^2}\right] = \frac{d}{dr_o}\left[\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{k}{h r_o^2}\right]$$

Equating the above to zero (at $r_o = r_{oc}$), gives

$$\frac{1}{r_{oc}^2} - \frac{2k}{h r_{oc}^3} = 0 \qquad \Longrightarrow \qquad \boxed{r_{oc} = \frac{2k}{h}}$$

Insulation Thickness vs. Heat Loss for Curved Surfaces



$$r_{oc} = \begin{cases} \dfrac{k}{h} & \text{for cylinder} \\[2ex] \dfrac{2k}{h} & \text{for sphere} \end{cases}$$

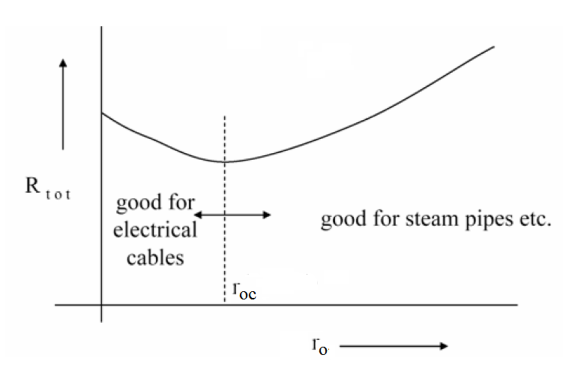
Heat loss from an insulated pipe varies with radius of insulation. Heat loss is maximum at critical radius. The thickness of insulation corresponding to critical radius of insulation is known as critical insulation thickness.

Insulation Thickness vs. Heat Loss for Curved Surfaces (contd..)

If outside radius of the pipe is smaller than the critical radius of the insulation, one does not have to insulate to save heat loss. But we do not want to leave hot pipes bare for safety reasons.

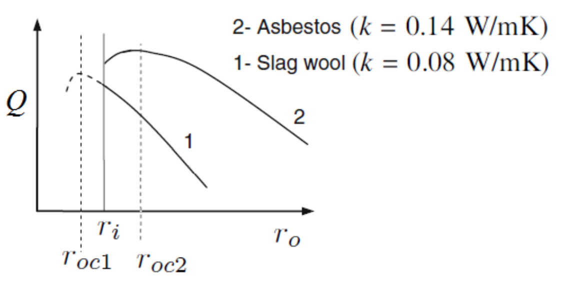
If the insulating material is chosen in such a way that $r_{oc} \leq r_i$, then any addition of insulation leads to decrease in heat loss.

Significance of Critical Radius of Insulation



Critical radius of insulation is a property of the insulating material and outside convection heat transfer coefficient. It is having significance only for radii smaller than about few cm.

On windy days, the external convection heat transfer coefficient is greater compared to calm days. Therefore critical radius of insulation will be greater on calm days.



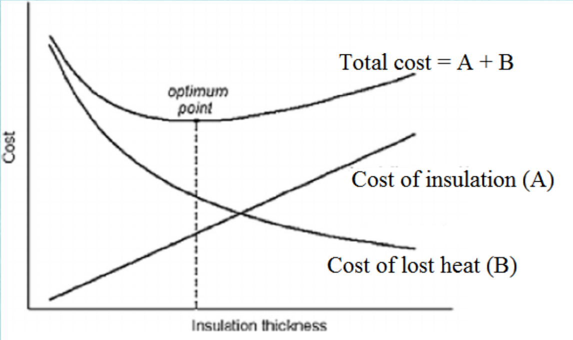
The value of the critical radius r_{oc} will be the largest when k is large and h is small.

Significance of Critical Radius of Insulation (contd..)

- Noting that the lowest value of h encountered in practice is about 5 W/(m^2.K) for the case of natural convection of gases, and that the thermal conductivity of common insulating materials is 0.05 W/(m.K), the largest value of the critical radius we are likely to encounter is $r_{oc} = k/h = 0.05/5 = 0.01 \text{ m} = 10 \text{ mm}$. This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection, often less than 1 mm, because of much larger h values associated with forced convection. Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.
- The radius of electric wires may be smaller than the critical radius. Therefore, the plastic electrical insulation may actually enhance the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.

Optimum Thickness of Insulation (from Economic Perspective)

The economic thickness of insulation depends on the first cost (insulating cost) and maintenance cost of insulation and annual value of heat loss, which depends on the cost of producing the steam and thermal conductivity of the lagging. Generally thicker insulation will represent higher owing costs and lower heat loss costs.



Questions to Think

Source: Cengel, Heat Transfer - a Practical Approach

1. A pipe is insulated such that the outer radius of the insulation is less than the critical radius. Now the insulation is taken off. Will the rate of heat transfer from the pipe increase or decrease for the same pipe surface temperature?
2. A pipe is insulated to reduce the heat loss from it. However, measurements indicate that the rate of heat loss has increased instead of decreasing. Can the measurements be right?
3. Consider a pipe at a constant temperature whose radius is greater than the critical radius of insulation. Someone claims that the rate of heat loss from the pipe has increased when some insulation is added to the pipe. Is this claim valid?
4. Consider an insulated pipe exposed to the atmosphere. Will the critical radius of insulation be greater on calm days or on windy days? Why?

Questions for Practice

1. A 0.083 inch diameter electrical wire at $115^{\circ}\mathrm{F}$ is covered by 0.02 inch thick plastic insulation with $k = 0.075$ (Btu/h.ft.$^{\circ}\mathrm{F}$). The wire is exposed to a medium at $50^{\circ}\mathrm{F}$, with a combined convection and radiation heat transfer coefficient of 2.5 Btu/(h.ft^2.$^{\circ}\mathrm{F}$). Determine if the plastic insulation on the wire will increase or decrease heat transfer from the wire. (Ans: insulation increases the heat transfer from the wire; as $r_{oc} > r_i$)
2. Calculate the critical radius of insulation for asbestos with $k = 0.17$ W/(m.K) surrounding a pipe and exposed to room air at $20^{\circ}\mathrm{C}$ with $h = 3$ W/(m^2.K). Calculate the heat loss from a 5 cm diameter pipe with a outer surface temperature of $100^{\circ}\mathrm{C}$ when covered with the critical radius of insulation and without insulation. (Ans: $r_{oc} = 5.67$ cm; $Q_{\text{with insulation}} = 47$ W/m; $Q_{\text{without insulation}} = 37.7$ W/m)

3. Modify the previous problem with fiberglass insulating material with $k = 0.04$ W/(m/K), and find the heat loss with 31.7 mm thickness of insulation. (Ans: $r_{oc} = 1.33$ cm; $Q_{\text{with insulation}} = 19.07$ W/m
4. Evaluate the thickness of rubber insulation necessary in the case of a 10 mm dia copper conductor to ensure max. heat transfer to the atmosphere, given the thermal conductivity of rubber as 0.155 W/(m.K) and the surface coefficient as 8.5 W/(m^2.K). Estimate the max. heat transfer rate per meter length of conductor if the temperature of rubber is not to exceed 65°C while the atmosphere is at 30°C. (Ans: 13.2 mm; 14.9 W/m)

5. A pipe of 20 mm inner diameter and 30 mm outer diameter is insulated with 35 mm thick insulation. The thermal conductivity of insulating material is 0.15 W/m.K and the convective heat transfer coefficient of outside air is 3 W/m^2.K. The temperature of bare pipe is 200°C and the ambient air temperature is 30°C. The heat transfer resistance of the pipe metal can be neglected.
   (a) Comment with reasoning about the heat transfer rates with and without insulation. (Ans: Heat transfer rate with insulation is higher than that without insulation. This is because of insulation thickness here corresponds to critical radius of insulation. $Q_{\text{without insulation}} = 48.07$ W/m; $Q_{\text{with insulation}} = 72.7$ W/m).
   (b) If the same insulating material is used, what is the minimum thickness above which there is a reduction in heat loss as compared to the bare pipe? (Ans: 352 mm; obtain this by equating $Q_{\text{without insulation}} = Q_{\text{with insulation}}$)
   (c) For optimum design, what conductivity of insulating material do you suggest for the conditions given in the problem? (Ans: 0.045 W/m.K; obtain this from $r_{oc} = r_i = k/h \implies k = r_i h$)
   (G-1993-21.b)

6. A steel pipe of outside diameter 30 cm carries steam and its surface temperature is 220°C. It is exposed to surroundings at 25°C. Heat is lost both by convection and radiation. The combined heat transfer coefficient has a value of 22 W/(m^2.K). Determine the heat loss for 1 m length. Check the economical merits of adding insulation pads of 7.5 cm thickness with thermal conductivity of 0.36 W/(m.K). The cost of heat is Rs. 1000 per GJ. The cost of insulation is Rs. 8000/m length. The unit is in operation for 200 hour/year. The capital cost should should be recovered in 2 years. After adding the insulation also the same convection and radiation prevail over the surface.

(Ans: Critical radius of insulation, $r_{oc} = k/h = 0.36/22 = 0.0164$ m; As $r_i > r_{oc}$, any addition of insulation leads to decrease of heat loss. Heat loss without insulation = 4043.8 W; Heat loss with insulation = 919.2 W; Reduction of heat loss due to insulation = $4043.8 - 919.2 = 3124.6$ W; Energy saved due to insulation per year = $3124.6 \times 200 \times 3600 = 2.25 \times 10^9$ J = 2.25 GJ; Cost of energy saved per year = Rs. 2.25×1000=R. 2250; Payback period = Investment / savings per year = 8000/2250 =3.56 year; which is higher than the recommended recovery period of 2 year. Hence, there is **no** economic incentive because of insulation. The pipe can be left bare.)