Heat Transfer

Conduction - Variable Thermal Conductivity

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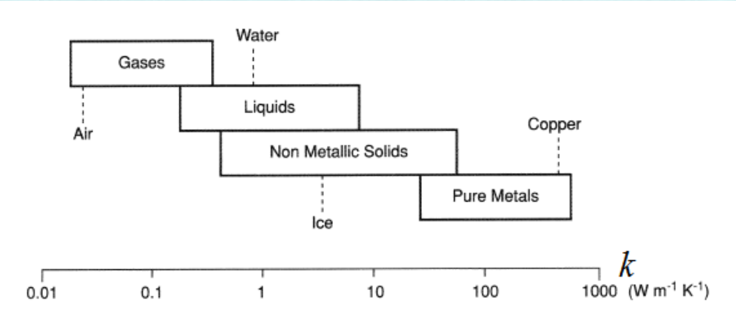
Objectives

- To understand the effect of variation of thermal conductivity of materials on heat transfer.

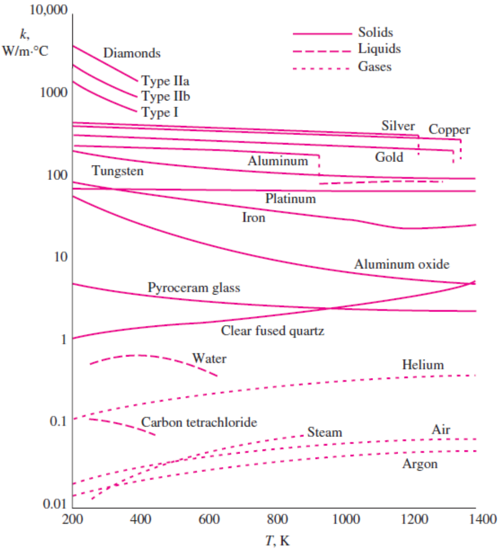
Outcome

- To obtain the equation for steady state temperature profile of variable thermal conductivity systems.

Thermal Conductivity of Materials



Thermal Conductivity of Materials



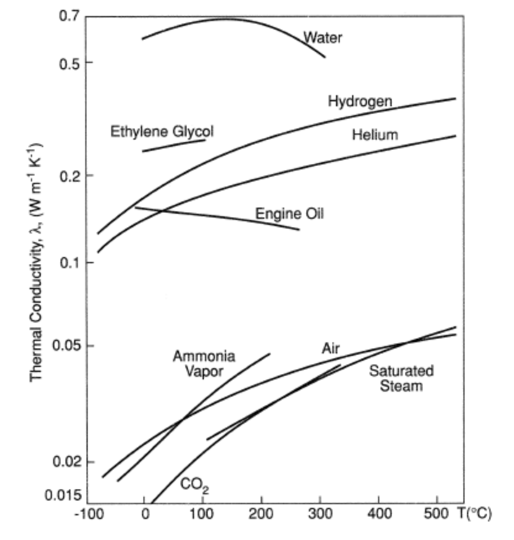
Thermal conductivities of materials vary with temperature

T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

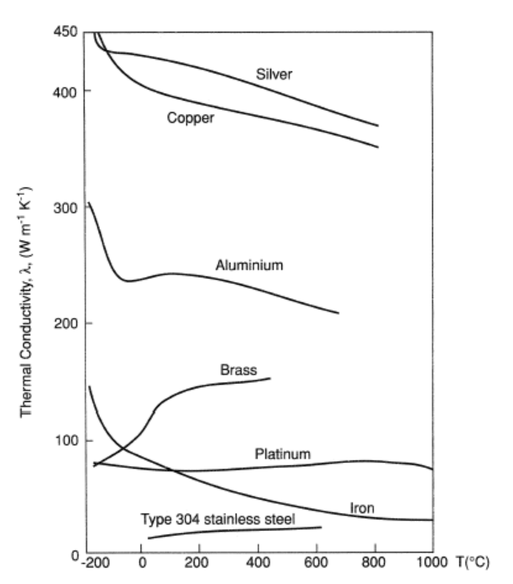
FIGURE 1–28
The variation of the thermal conductivity of various solids, liquids, and gases with temperature

Cengel, Heat Transfer - a Practical Approach, 2nd Edition.

Thermal Conductivity of Fluids



Thermal Conductivity of Solids



- Thermal conductivity of fluids increases with the increase in temperature. The exception is water, which exhibits increasing k up to about $150^\circ\mathrm{C}$ and decreasing k thereafter. Water has the highest thermal conductivity of all liquids except for liquid metals.
- Thermal conductivity of solids (in general) decreases with the increase in temperature.

Thermal Conductivity Variation with T

- The thermal conductivity of a material, in general, varies with temperature. An average value for the thermal conductivity is used when the variation is mild.
- When the variation of thermal conductivity conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1}$$

- The variation in thermal conductivity of a material can often be approximated as a linear function and expressed as

$$\boxed{k(T) = k_o(1 + \beta T)}$$

where β is the temperature coefficient of thermal conductivity.

Accounting for Variation of k in Steady Heat Transfer

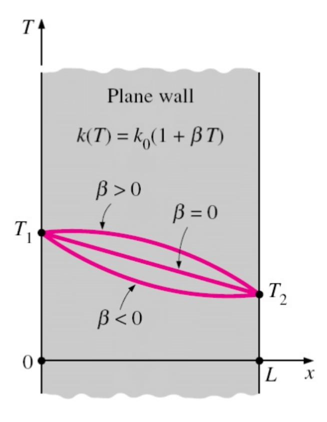
$$Q_{\text{flat}} = k_{\text{avg}} A \frac{T_1 - T_2}{L} = \frac{A}{L} \int_{T_2}^{T_1} k(T)\, dT$$

$$Q_{\text{cylinder}} = 2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_1/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)} \int_{T_2}^{T_1} k(T)\, dT$$

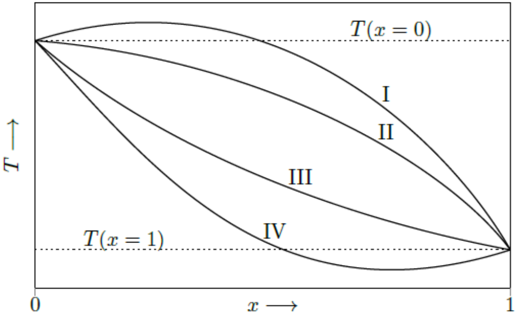
$$Q_{\text{sphere}} = 4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1 r_2}{r_2 - r_1} \int_{T_2}^{T_1} k(T)\, dT$$

Temperature Profile for Plane Wall

For a plane wall the temperature varies linearly during steady one-dimensional heat conduction when the thermal conductivity is constant. This is no longer the case when the thermal conductivity changes with temperature.



Consider a solid block of unit thickness for which the thermal conductivity decreases with an increase in temperature. The opposite faces of the block are maintained at constant but different temperatures: $T(x=0) > T(x=1)$. Heat transfer is by steady state conduction in x-direction only. There is no source or sink of heat inside the block. In the figure below, identify the correct temperature profile in the block. (G-2015-51)



(a) I (b) II (c) III (d) IV

(c) ✓ *Explanation:* Steady state heat flux through a flat wall is given by

$$q = -k\frac{dT}{dx} = \text{ constant}$$

For constant k, we get $|dT/dx|$ is constant. i.e., a linear slope.

Given: k decreases with increase in temperature; i.e., $k \propto 1/T$.

In the direction of heat flow, T decreases with x. Hence, k increases with x. Increased k leads to the requirement of decreased $|dT/dx|$, so as to make q constant.

Likewise, in the inlet T is high, and hence k is low. Low k leads to higher $|dT/dx|$ so as to make q constant.

These lead to to requirement of higher slope of dT/dx near high T regions and lower slope of dT/dx in the low T regions. i.e., slope of dT/dx curve goes from high to low with increase in x. Curve III represents this behavior.

Curve II is opposite of curve III. i.e., curve II represents the behavior of system for which $k \propto T$.

Maxima or minima in dT/dx are possible only for systems with heat generation or sink. i.e., curve I is applicable for system with heat generation; and curve IV is applicable for system with heat sink. □