Heat Transfer

Convection - Correlations for Heat Transfer Coefficient

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September 25, 2019

Objectives

- To give an overview about the variables of importance in convective heat transfer

Outcome

- To understand the significance of various dimensionless numbers in convection.
- To understand the various empirical correlations used and their specific conditions of usage in convective heat transfer.

Hydrodynamic Boundary Layer

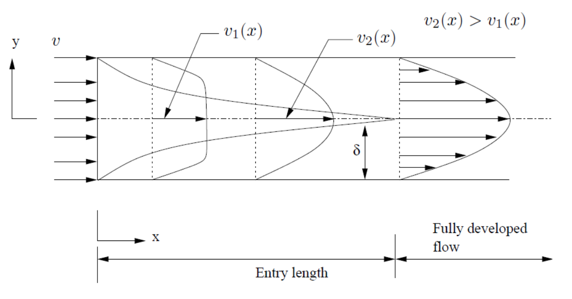


Figure : Flow development in pipe

Entry Length:

Laminar flow: $$\left(\frac{L}{D}\right)_{e,h} \approx 0.05 \text{ Re}$$

Turbulent flow: $$\left(\frac{L}{D}\right)_{e,h} > 10$$

Critical Reynolds Number

Critical Reynolds number approximates the location where the flow transitions from laminar to turbulent flow.

$$\mathrm{Re}_{x,c} \approx 10^5 \qquad \text{external (flat plate) flow}$$

$$\mathrm{Re}_{D,c} \approx 10^3 \qquad \text{internal (duct) flow}$$

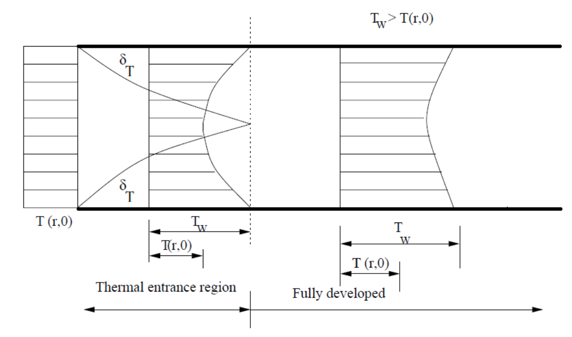


Figure : Thermally developing flow

Thermal Entry Length: for laminar flow,

$$\left(\frac{L}{D}\right)_{e,t} \approx 0.05 \, \text{Re Pr}$$

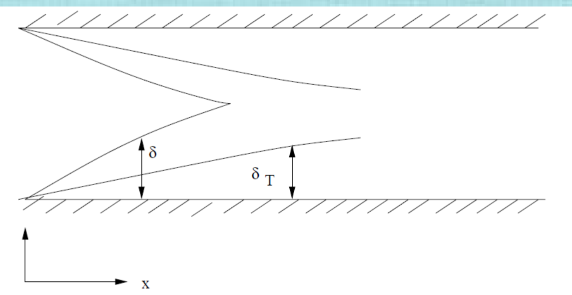


Figure : Hydrodynamic and thermal boundary layer thickness for $\mathrm{Pr} \gg 1$

Comparing thermal boundary layer with hydrodynamic boundary layer, it can be said that if $\mathrm{Pr} > 1$, hydrodynamic boundary layer grows more rapidly.

$$\mathrm{Pr} > 1, \quad L_{e,h} < L_{e,t} \quad \Longrightarrow \quad \delta > \delta_T \quad \text{at any section}$$

Inverse is true for $\mathrm{Pr} < 1$.

It is significant that the Prandtl number for most gases are sufficiently close to unity, and the hydrodynamic and thermal boundaries are of similar extent.

When $\mathrm{Pr} > 1$, $\delta > \delta_t$, and when $\mathrm{Pr} < 1$, $\delta < \delta_t$. High viscosity leads to a thick momentum boundary layer, and high thermal diffusivity leads to a thick thermal boundary layer.

Thermally Fully Developed Conditions

Since the existence of convective heat transfer between the surface and the fluid dictates that the fluid temperature must continue to change with x, one might question whether fully developed conditions ever can be reached. The situation is certainly different from the hydrodynamic case, for which $(\partial v/\partial x) = 0$ in the fully developed region. In contrast, if there is heat transfer, $(\partial T/\partial x)$ at any radius is not zero. Introducing a dimensionless temperature $(T_w - T)/(T_w - T_m)$, condition for which this ratio becomes independent of x are known to exist. Although the temperature profile $T(r)$ continues to change with x, the relative shape of the profile does not change and the flow is said to be fully developed. Instead of $(\partial T/\partial x) = 0$, the condition is

$$\boxed{\frac{d}{dx}\left[\frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}\right] = 0}$$

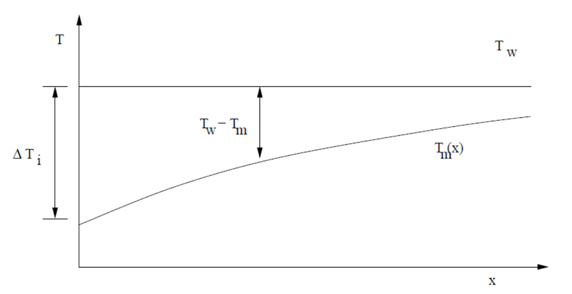
i.e., $(T_w - T)$ changes in the same way as $(T_w - T_m)$.

Tube Surface Conditions

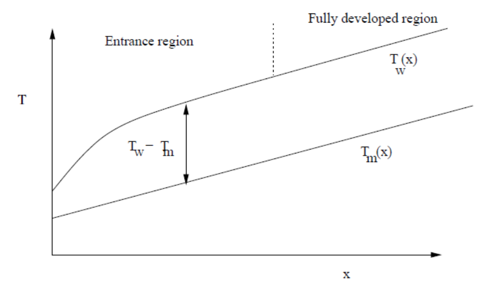
Tube surface condition: either uniform wall temperature condition (UWT) condition, or a uniform wall heat flux condition (UWH).
For $T_w =$ **constant, i.e.,** $(dT_w/dx) = 0$: If a phase change were occurring at outer surface.

For $q_w = h(T_w - T_m) =$ **constant**: If the tube wall were heated electrically or, if the outer surface were uniformly irradiated.

Constant Wall Temperature



Constant Wall Heat Flux

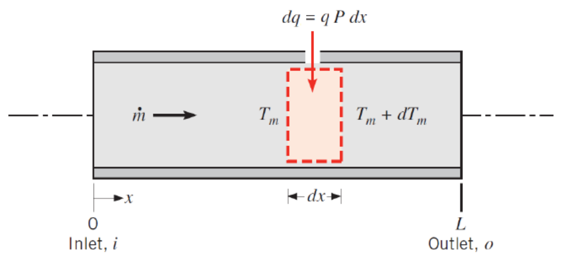


By assumption, $q_w = h(T_w - T_m) =$ constant.
For thermally developed flow, $h =$ constant. Then,

$$\frac{d}{dx}(q) = \frac{d}{dx}\left[h(T_w - T_m)\right] = 0$$

i.e.,

$$0 = h\left(\frac{dT_w}{dx} - \frac{dT_m}{dx}\right) \qquad \Longrightarrow \qquad \boxed{\frac{dT_w}{dx} = \frac{dT_m}{dx}}$$



$$\frac{dT_m}{dx} = \frac{q_w P}{\dot{m} C_P} = \text{ constant}$$

where P is the perimeter of wall. Integrating from $x = 0$, we get

$$T_m(x) = T_{m,i} + \frac{q_w P}{\dot{m} C_P} x \qquad \Longrightarrow \qquad \text{Linear variation}$$

T_w is constant.

$$\frac{T_w - T_m(x)}{T_w - T_{m,i}} = \exp\left[-\left(\frac{Ph}{\dot{m}C_P}\right)x\right]$$

$$q = hA\Delta T_{\text{lm}}$$

where

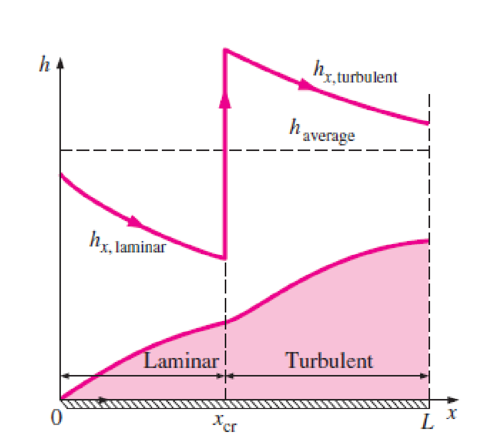
$$\Delta T_{\text{lm}} = \text{Log Mean Temperature Difference (LMTD)} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}$$

Note: in all equations evaluate fluid properties at the film temperature (T_f) defined as the arithmetic mean of the surface and free-stream temperatures unless otherwise stated.

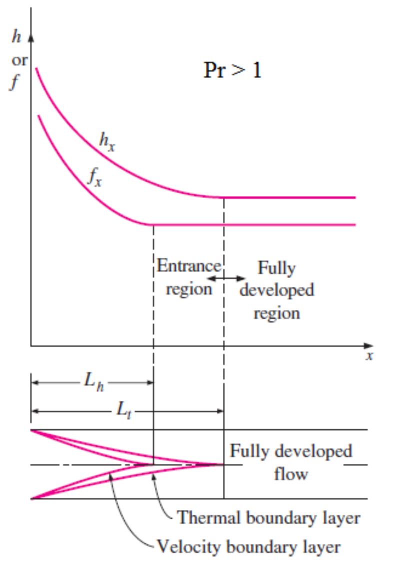
$$T_f = \frac{T_S + T_\infty}{2} = \frac{T_w + T_m}{2}$$

Local and Average Heat Transfer Coefficients

$$\boxed{h = \frac{1}{L}\int_0^L h_x\, dx}$$



Local and Average Heat Transfer Coefficients (contd..)

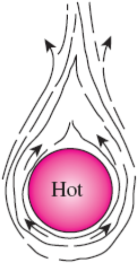


- Friction factor and the heat transfer coefficient remain constant in fully developed laminar or turbulent flow, since the velocity and normalized temperature profiles do not vary in the flow direction.
- Laminar and turbulent boundary layers have different heat transfer characteristics turbulent mixing typically increases heat transfer.

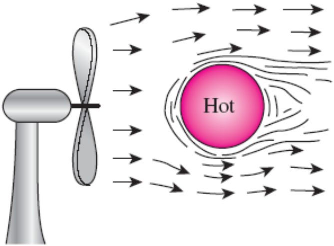
Mixed Convection

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient $\mathrm{Gr}/\mathrm{Re}^2$:

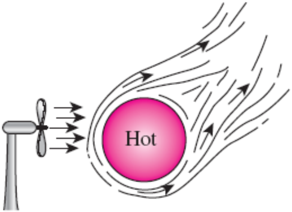
- Natural convection effects are negligible if $\mathrm{Gr}/\mathrm{Re}^2 \ll 1$.
- Free convection dominates and the forced convection effects are negligible if $\mathrm{Gr}/\mathrm{Re}^2 \gg 1$.
- Both effects are significant and must be considered if $\mathrm{Gr}/\mathrm{Re}^2 = 1$ (mixed convection).



(*a*) Natural convection ($Gr_L / Re_L^2 >> 1$)



(*b*) Forced convection ($Gr_L / Re_L^2 << 1$)



(*c*) Mixed convection ($Gr_L / Re_L^2 \approx 1$)

FIGURE

The relative importance of convection heat transfer regimes for flow near a hot sphere.

Dimensionless Numbers

Dimensionless number	Formula	Importance
Reynolds number (Re)	$\frac{Dv\rho}{\mu}$	$\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}}$ $\mathrm{Re} < 2000$, flow is laminar.
Prandtl number (Pr)	$\frac{C_p\mu}{k}$	$\mathrm{Pr} = \frac{(\mu/\rho)}{(k/\rho C_p)} = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}}$
Nusselt number (Nu)	$\frac{hL}{k}$	$\mathrm{Nu} = \frac{h\Delta T}{k\Delta T/L} = \frac{\text{heat transfer by convection}}{\text{heat transfer by conduction}}$ where ΔT is temperature difference between the wall surface and fluid temperatures. Nusselt number equal to unity implies that there is no convection – the heat transfer is purely by conduction across the layer of thickness L.
Grashof number (Gr)	$\frac{g\beta L^3(T_w - T_\infty)}{(\mu/\rho)^2}$	$\mathrm{Gr} = \frac{(\text{buoyancy force})^2}{(\text{viscous force})^2}$ β = volumetric coefficient of thermal expansion of the fluid (unit: $1/°\mathrm{C}$)
Parameter $\mathrm{Gr}/\mathrm{Re}^2$	$\frac{\mathrm{Gr}}{\mathrm{Re}^2}$	$\frac{\mathrm{Gr}}{\mathrm{Re}^2} = \frac{(\text{buoyancy force})^2}{(\text{inertial force})^2}$ It is a measure of the relative importance of the free convection in relation to forced convection. If $(\mathrm{Gr}/\mathrm{Re}^2) \ll 1$, flow is primarily by forced convection.

Dimensionless Numbers (contd..)

Dimensionless number	Formula	Importance
Rayleigh number (Ra)	Gr.Pr	Used in place of Grashof number to correlate heat transfer in free convection.
Peclet number (Pe)	Re.Pr	Used in place of Reynolds number to correlate heat transfer in forced convection.
Stanton number (St)	$\frac{\mathrm{Nu}}{\mathrm{Re.Pr}}$	$\mathrm{St} = \frac{h\Delta T}{\rho C_p v \Delta T}$ The numerator represents the heat flux to the fluid, and denominator represents the heat transfer capacity of the fluid flow.
Graetz number (Gz)	$\frac{\mathrm{Re} \cdot \mathrm{Pr}}{L/D}$	Graetz number is a correlating parameter in thermally developing flow.

Laminar flow:

$$Nu_x = \frac{h_x x}{k} = 0.332\ \mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

Turbulent flow:

$$Nu_x = \frac{h_x x}{k} = 0.03\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3} \qquad \begin{array}{c} 0.6 \leq \mathrm{Pr} \leq 60 \\ 5 \times 10^5 \leq \mathrm{Re}_x \leq 10^7 \end{array}$$

Note:
For laminar flow, h_x is proportional to $\mathrm{Re}_x^{0.5}$ and thus to $x^{-0.5}$.
For turbulent flow, h_x is proportional to $x^{-0.2}$.

Laminar flow:

$$Nu_L = \frac{hL}{k} = 0.664\ \mathrm{Re}_L^{0.5}\ \mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

Forced Convection - Laminar Flow through Circular Pipe

Convection heat transfer associated with laminar flow in a circular pipe ($\mathrm{Re} < 2100$) is less common than with turbulent flow. The correlations are simple, with Nusselt number being constant for fully developed flow (both hydrodynamically and thermally).

For uniform wall heat flux:	$\mathrm{Nu} = 4.36$
For uniform wall temperature:	$\mathrm{Nu} = 3.66$

The L/D ratio for the entrance length required to reach fully developed flow is larger for laminar flow than for turbulent flow.

Forced Convection - for Turbulent Flow inside Circular Pipe

Dittus-Boelter equation (1930)

$$\boxed{\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}}$$

where

$$n = \begin{cases} 0.3 \text{ for cooling of the fluid } (T_s < T_\infty) \\ 0.4 \text{ for heating of the fluid } (T_s > T_\infty) \end{cases}$$

The Dittus-Boelter equation is valid for smooth pipes and for

$$0.6 \leq \mathrm{Pr} \leq 160 \qquad \mathrm{Re} \geq 10,000 \qquad \frac{L}{D} \geq 10$$

Forced Convection - for Turbulent Flow inside Circular Tube (contd..)

The Dittus-Boelter equation is recommended only for rather small temperature differences between the bulk fluid and the pipe wall. A few years later, in 1936, Sieder and Tate proposed the following equation to accommodate larger temperature differences:

$$\boxed{\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3}\ (\mu/\mu_w)^{0.14}}$$

where

$$\begin{aligned} \mu &= \text{viscosity of the fluid at the fluid bulk temperature} \\ \mu_w &= \text{viscosity of the fluid at the pipe wall temperature} \end{aligned}$$

Valid for:

$$0.7 \leq \mathrm{Pr} \leq 16,700 \qquad \mathrm{Re} \geq 10,000 \qquad \frac{L}{D} \geq 10$$

Both the Dittus-Boelter and Seider-Tate equations are still in widespread use.

Free Convection

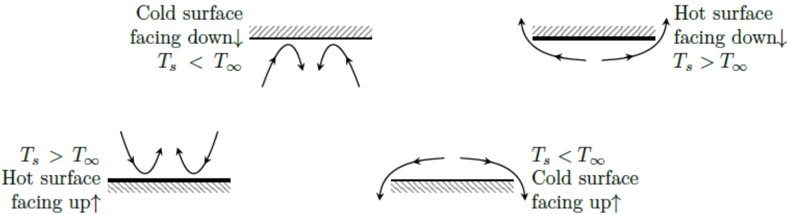


Figure 9: Flow Patterns for Various Conditions in Free Convection (T_s = temperature of plate surface; T_∞ = temperature of bulk fluid)

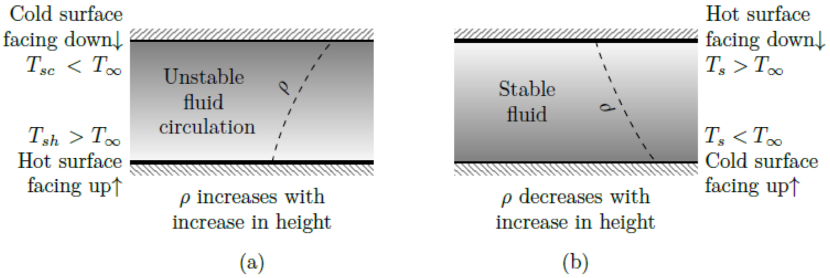


Figure 10: Fluid Contained between Two Horizontal Plates

- **Isothermal Vertical Plate**

$$Nu = \begin{cases} 0.59\,\mathrm{Ra}^{1/4} & 10^4 < \mathrm{Ra} < 10^9 \quad \text{(laminar flow)} \\ 0.1\,\mathrm{Ra}^{1/3} & 10^9 < \mathrm{Ra} < 10^{13} \quad \text{(turbulent flow)} \end{cases}$$

- **Isothermal Horizontal Plate**
  - Hot plate facing upwards:

$$Nu = \begin{cases} 0.54\,\mathrm{Ra}^{1/4} & 10^4 < \mathrm{Ra} < 10^7 \\ 0.15\,\mathrm{Ra}^{1/3} & 10^7 < \mathrm{Ra} < 10^{11} \end{cases}$$

  - Hot plate facing downwards:

$$Nu = 0.27\,\mathrm{Ra}^{1/4} \qquad 10^7 < \mathrm{Ra} < 10^{11}$$