Heat Transfer

Convection - Solved Problems

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Example 1:Thermally Developing Flow

Consider the flow of a gas with density 1 kg/m^3, viscosity 1.5×10^{-5} kg/(m.s), specific heat $C_p = 846$ J/(kg.K) and thermal conductivity $k = 0.01665$ W/(m.K), in a pipe of diameter $D = 0.01$ m and length $L = 1$ m, and assume the viscosity does not change with temperature. The Nusselt number for a pipe with (L/D) ratio greater than 10 and Reynolds number greater than 20000 is given by

$$\text{Nu} = 0.026\ \text{Re}^{0.8}\ \text{Pr}^{1/3}$$

While the Nusselt number for a laminar flow for Reynolds number less than 2100 and (Re Pr D/L) < 10 is

$$\text{Nu} = 1.86\ [\text{Re Pr } (D/L)]^{1/3}$$

If the gas flows through the pipe with an average velocity of 0.1 m/s, the heat transfer coefficient is (GATE-2005)

(a) 0.68 W/(m^2.K) (b) 1.14 W/(m^2.K)
(c) 2.47 W/(m^2.K) (d) 24.7 W/(m^2.K)

Solved Problems (contd..)

Solution:

From the given data,

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = \frac{0.01 \times 0.1 \times 1}{1.5 \times 10^{-5}} = 66.7$$

$$\mathrm{Pr} = \frac{C_p\mu}{k} = \frac{846 \times 1.5 \times 10^{-5}}{0.01665} = 0.76$$

$$\mathrm{Re} \cdot \mathrm{Pr} \cdot (D/L) = 66.7 \times 0.76 \times (0.01/1) = 0.507$$

Hence, using the expression of Nu valid for this condition, we get

$$\mathrm{Nu} = 1.86\ [\mathrm{Re\ Pr}\ (D/L)]^{1/3} = 1.86 \times (0.507)^{1/3} = 1.483$$

By definition, $\mathrm{Nu} = hD/k$. Therefore,

$$h = \frac{\mathrm{Nu} \times k}{D} = \frac{1.483 \times 0.01665}{0.01} = 2.47\ \mathrm{W/(m^2.K)} \qquad \textbf{(c)}\ \checkmark$$

Solved Problems (contd..)

Example 2:Heat Transfer Coefficient

Hot liquid is flowing at a velocity of 2 m/s through a metallic pipe having an inner diameter of 3.5 cm and length 20 m. The temperature at the inlet of the pipe is 90°C. Following data is given for liquid at 90°C:

$$Density = 950\ kg/m^3$$
$$Specific\ heat = 4.23\ kJ/kg.^\circ C$$
$$Viscosity = 2.55 \times 10^{-4}\ kg/m.s$$
$$Thermal\ conductivity = 0.685\ W/m.^\circ C$$

The heat transfer coefficient (in $kW/m^2.^\circ C$) inside the tube is (GATE-2008)

(a) 222.22 (b) 111.11 (c) 22.22 (d) 11.11

Solved Problems (contd..)

Solution:

From Dittus-Boelter relation, we have $\text{Nu} = 0.023\,\text{Re}^{0.8}\text{Pr}^{0.33}$.
For the given data,

$$\text{Re} = \frac{Dv\rho}{\mu} = \frac{0.035 \times 2 \times 950}{2.55 \times 10^{-4}} = 260784$$

$$\text{Pr} = \frac{C_P\mu}{k} = \frac{4.23 \times 1000 \times 2.55 \times 10^{-4}}{0.685} = 1.575$$

$$\text{Nu} = 0.023 \times (260784)^{0.8} \times (1.575)^{0.33} = 575.2$$

i.e., $$\frac{hD}{k} = 575.2$$

$$\Longrightarrow \quad h = 575.2 \times \frac{0.675}{0.035}$$

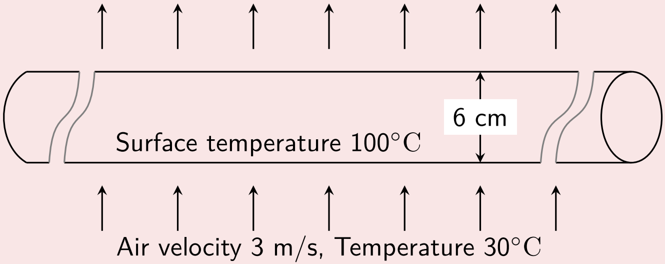
$$= 11,257\ \text{W/m}^2.^\circ\text{C} = 11.3\ \text{kW/m}^2.^\circ\text{C}$$ **(d)** ✓

Solved Problems (contd..)

Example 3:Heat Transfer Rate

Air is flowing at a velocity of 3 m/s perpendicular to a long pipe as shown in the figure below. The outside diameter of the pipe is $d = 6$ cm and temperature at the outside surface of the pipe is maintained at $100^\circ\mathrm{C}$. The temperature of the air far from the tube is $30^\circ\mathrm{C}$.
Data for air: Kinematic viscosity, $\nu = 18 \times 10^{-6}$ m^2/s;
Thermal conductivity, $k = 0.03$ W/(m.K)

Using the Nusselt number correlation: $\mathrm{Nu} = \dfrac{hD}{k} = 0.024 \times \mathrm{Re}^{0.8}$, the rate of heat loss per unit length (W/m) from the pipe to air (up to one decimal place) is ________ (GATE-2015-50)



Solved Problems (contd..)

Solution:

$$\begin{aligned}
\mathrm{Re} &= Dv\rho/mu = Dv/(\mu/\rho) = Dv/\nu \\
&= 6\times 10^{-2}\times 3/(18\times 10^{-6}) = 10000 \\
\mathrm{Nu} &= 0.024\times \mathrm{Re}^{0.8} = 0.024\times (10000)^{0.8} = 38.037 = \frac{hD}{k} \\
h &= 38.037\times \frac{k}{D} = 38.037\times \frac{0.03}{6\times 10^{-2}} = 19.02\ \mathrm{W/m^2.K} \\
Q &= hA\Delta T = h(\pi DL)\Delta T \\
Q/L &= h(\pi D)\Delta T \\
&= 19.02\times \pi\times 6\times 10^{-2}\times (100-30) = 251\ \mathrm{W/m}
\end{aligned}$$

Example 4:Heat Transfer by Convection from a Sphere

A 200 W heater has a spherical casing of diameter 0.2 m. The heat transfer coefficient for conduction and convection from the casing to the ambient air is obtained from $\text{Nu} = 2 + 0.6\text{Re}^{1/2}\text{Pr}^{1/3}$, with $\text{Re} = 10^4$ and $\text{Pr} = 0.69$. The temperature of the ambient air is 30°C and the thermal conductivity of air is $k = 0.02$ W/m.K.

1. Find the heat flux from the surface at steady state.
2. Find the steady state surface temperature of the casing.
3. Find the temperature of the casing at steady state for stagnant air. Why is this situation physically infeasible? (GATE-2001)

Solution:

Heat flux (q) from the surface is given by

$$q = \frac{Q}{A} = \frac{Q}{4\pi r^2} = \frac{200}{4 \times \pi \times 0.1^2} = 1591.5 \text{ W/m}^2$$

Nusselt number (Nu) for convection:

$$Nu = 2 + 0.6\text{Re}^{1/2}\text{Pr}^{1/3} = 2 + 0.6 \times (10000)^{1/2} \times (0.69)^{1/3} = 55.02$$

Convective heat transfer coefficient (h):

$$h = \frac{\text{Nu} \cdot k}{D} = \frac{55.02 \times 0.02}{0.2} = 5.502\text{W/(m}^2\text{.K)}$$

For heat transfer by convection,

$$q = h(T - T_\infty)$$

Substituting the known quantities, we get

$$1591.5 = 5.502 \times (T - 30)$$
$$\Longrightarrow \quad T = 319.3^\circ\mathrm{C}$$

i.e., The steady state surface temperature of spherical casing is 319.3°C. If the air is stagnant, then Re = 0. This leads to Nu = 2. Therefore, the heat transfer coefficient for this condition becomes,

$$h = \frac{\mathrm{Nu} \cdot k}{D} = \frac{2 \times 0.02}{0.2} = 0.2\ \mathrm{W/(m^2.K)}$$

Using this value of h, for the heat flux of 1591.5 $\mathrm{W/m^2}$, we get

$$q = h(T - T_\infty)$$
$$1591.5 = 0.2 \times (T - 30)$$
$$\Longrightarrow \quad T = 7987.5^\circ\mathrm{C}$$

Solved Problems (contd..)

This situation (the condition of stagnant air) cannot be maintained for a long time, as explained below:
Surface temperature of 7987.5°C leads to reducing the density of nearby air sharply, as $\rho \propto T^{-1}$ (as from the ideal gas relation, we have $\rho \propto P/(RT)$). This leads to setting up of convection currents, and hence the increase of Nusselt number, thereby reducing the surface temperature.

Example 5:Momentum & Heat Transfer Analogy

Air flows through a smooth tube, 2.5 cm diameter and 10 m long, at 37°C. If the pressure drop through the tube is 10000 Pa, estimate

(a) the air velocity through the tube and the friction factor

(b) the heat transfer coefficient using Colburn Analogy [$j_H = (\text{St})(\text{Pr})^{0.67}$], where St is the Stanton Number and Pr is the Prandtl Number.

Gas constant, $R = 82.06$ cm^3.atm/mol.K. Darcy friction factor $= 0.184/\text{Re}^{0.2}$. Other relevant properties of air under the given conditions: viscosity $= 1.8 \times 10^{-5}$ kg/m.s, density $= 1.134$ kg/m^3, specific heat capacity, $C_p = 1.046$ kJ/kg.°C, thermal conductivity $= 0.028$ W/m.°C. (GATE-2002)

Solution:

Pressure drop due to friction is related to velocity as

$$\Delta P = \frac{2fL\rho v^2}{D} \tag{1}$$

Given: $f =$ Darcy friction factor $= 0.184/\mathrm{Re}^{0.2}$.

$$\text{Darcy friction factor } = 4 \times \text{ Fanning friction factor}$$

In Eqn.(1), f denotes Fanning friction factor. Therefore,

$$f = 0.25 \times 0.184/\mathrm{Re}^{0.2} = 0.046/\mathrm{Re}^{0.2}$$

Expanding,

$$f = \frac{0.046}{(Dv\rho/\mu)^0.2} = \frac{0.046\mu^{0.2}}{(Dv\rho)^{0.2}} \tag{2}$$

Substituting this in Eqn.(1),

$$\begin{aligned}\Delta P &= \frac{2\times 0.046\times \mu^{0.2}L\rho v^2}{D^{1.2}v^{0.2}\rho^{0.2}} \\ &= \frac{0.092\mu^{0.2}L\rho^{0.8}v^{1.8}}{D^{1.2}}\end{aligned}$$

Substituting for the known quantities,

$$10000 = \frac{0.092\times(1.8\times 10^{-5})^{0.2}\times 10\times(1.134)^{0.8}\times v^{1.8}}{(2.5\times 10^{-2})^{1.2}}$$

Solving, v = air velocity through the tube = 47.6 m/s.
From Eqn.(2),

$$f = \frac{0.046\times(1.8\times 10^{-5})^{0.2}}{(2.5\times 10^{-2}\times 22.02\times 1.134)^{0.2}} = 0.0049$$

Solved Problems (contd..)

By Colburn analogy,

$$j_H = (\text{St})(\text{Pr})^{0.67} = \frac{f}{2}$$

where

$$\begin{aligned} \text{St} &= \text{Stanton number} = \text{Nu}/(\text{Re} \cdot \text{Pr}) = h/(\rho C_p v) \\ \text{Pr} &= \text{Prandtl number} = C_p \mu / k \\ f &= \text{Fanning friction factor} \end{aligned}$$

Therefore,

$$\frac{h}{1.134 \times 1046 \times 47.6} \times \left(\frac{1046 \times 1.8 \times 10^{-5}}{0.028} \right)^{0.67} = \frac{0.0049}{2}$$

Solving, h = heat transfer coefficient = 180.5 W/m^2.K

A fluid flows over a heated horizontal plate maintained at temperature T_w. The bulk temperature of the fluid is T_∞. The temperature profile in the thermal boundary layer is given by:

$$T = T_w + (T_w - T_\infty)\left[\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 - \frac{3}{2}\left(\frac{y}{\delta_t}\right)\right], \qquad 0 \le y \le \delta_t$$

Here, y is the vertical distance from the plate, δ_t is the thickness of the thermal boundary layer and k is the thermal conductivity of the fluid.

The local heat transfer coefficient is given by (G-2017-40)

(a) $\frac{k}{2\delta_t}$ (b) $\frac{k}{\delta_t}$ (c) $\frac{3}{2}\frac{k}{\delta_t}$ (d) $2\frac{k}{\delta_t}$

(c) ✓ *Explanation:* By definition,

$$q = -k \left. \frac{\partial T}{\partial y} \right|_{y=0} = h(T_w - T_\infty)$$

Hence,

$$h = \frac{-k \left. \frac{\partial T}{\partial y} \right|_{y=0}}{T_w - T_\infty}$$

From the given relation of T,

$$T = T_w + (T_w - T_\infty) \frac{1}{2} \frac{y^3}{\delta_t^3} - (T_w - T_\infty) \frac{3}{2} \frac{y}{\delta_t}$$

$$\frac{\partial T}{\partial y} = (T_w - T_\infty) \frac{3}{2} \frac{y^2}{\delta_t^3} - (T_w - T_\infty) \frac{3}{2} \frac{1}{\delta_t}$$

$$\left. \frac{\partial T}{\partial y} \right|_{y=0} = -(T_w - T_\infty) \frac{3}{2} \frac{1}{\delta_t} \qquad \Longrightarrow \qquad h = \frac{-k \left. \frac{\partial T}{\partial y} \right|_{y=0}}{T_w - T_\infty} = \frac{3}{2} \frac{k}{\delta_t}$$